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Summary of my PhD thesis in mathematics entitled

"Excluding and constructing of exotic group actions on spheres"

In their article [4], Wu-Chung Hsiang and Wu Yi Hsiang write on pages 224 and 231 the following.

"Due to the existence of natural linear actions on Euclidean spaces, spheres and disks, it is quite fair to say that they are the best testing spaces in the study of differentiable transformation groups (...) We share the prevailing conviction that the study of differentiable actions on these best testing spaces is probably still the most important topic in transformation groups."

The thesis concerns exotic smooth actions of finite groups on manifolds. We focus on actions on spheres with one and two fixed points. The exoticism means that the actions are not equivalent to linear ones. An important problem is, on one hand, exlcuding of the exotic actions, and, on the other hand, the constructions of them. For two fixed point actions of a finite group G on a sphere, we focus on these cases where the tangent spaces at the two fixed points have non-isomorphic $\mathbb{R}G$-module structures.

The first subject of our research are exclusions of smooth one fixed point actions of finite groups on spheres. We develop a strategy of excluding of such actions on spheres of a given dimension. The strategy relies on homological properties of the fixed point data and intersection theory. We provide new algebraic conditions, sufficient to exclude one fixed point actions. We present an algorithm which, by verifying the appropriate sufficient conditions (both described in this thesis, and the conditions obtained earlier by Morimoto and Tamura [6] and Borowiecka and the author [1,2]), allows us to exclude the actions in question. This algorithm, implemented in GAP [3], provides new exclusion results.

This thesis is also concerned with two fixed point actions on spheres having non-isomorphic $\mathbb{R}G$-module structures on the tangent spaces at the fixed points, which are defined by differentiation of the action. The question about the existence of such actions was raised by Smith [7] who asked whether for a finite group G acting smoothly on a sphere with exactly two fixed points, the $\mathbb{R}G$-module structures induced on the tangent spaces at the two fixed points, are always isomorphic to each other. There is a conjecture of Laitinen [5] which predicts the negative answer to the Smith question for groups satisfying certain algebraic conditions. Although not true in general, the conjecture holds for many families of finite groups. Still, the Laitinen Conjecture remains unsettled for various families of groups. Our main result of this part is indicating a new infinite family of finite groups for which the Laitinen Conjecture holds.

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