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Review of the thesis  
“Full nonassociative Lambek calculus with modalities and its  
applications in type grammars”  
by Zhe Lin

**Background** Lambek calculus and its extensions have numerous applications in computational linguistics, because type grammars based on them are a very useful formalism in natural language syntax. Namely, the logical connectives of Lambek calculus are noncommutative product  $\cdot$  and two corresponding residuals  $\backslash$  and  $/$  (left and right divisions, respectively).

For example,  $A\backslash B$  is the type of a phrase that results in a phrase of type  $B$  when preceded (on the left) by a phrase of type  $A$ ,  $B/A$  is the type of a phrase that results in a phrase of type  $B$  when followed (on the right) by a phrase of type  $A$ , and  $\cdot$  serves as a phrase composition.

Also, it is argued in the literature that type grammars based on nonassociative Lambek calculus (NL) are a more appropriate formalism than those based on the associative one, because the latter appear to be too strong. This is because the latter accept all possible phrase structures on the accepted strings, (but some desirable type relations are not derivable in these type grammars).

On the other hand, type grammars based on NL calculus are a weak grammar formalisms for natural languages. Enriching them with modal and ordinary logical connectives and non-logical assumptions allows one to mark morphosyntactic distinctions and describe lexical ambiguity. In particular, the former is marked by a modal operator and the latter is described by additive connectives.

In addition, Lambek calculus, as a pure substructural logic, is very attractive and important in its own right, and its semantics, decision properties, complexity, substructural extensions with modal logic, structural extensions with intuitionistic and classical logics, and relation to formal languages (context-free grammars) have been widely studied in the literature. There are many books on Lambek calculus. Also, Google scholar gives more than five thousand papers on the above topics.

All this is a solid motivation for the submitted thesis.

**Description of the thesis and the thesis contribution** The thesis deals with NL extended with modal and classical propositional connectives and the corresponding axioms and/or rules of inference. In particular, the modal connectives also satisfy the residuation law. Extensions of the above calculi with proper axioms (assumptions) are also studied in the thesis.

The thesis consists of seven chapters which are as follows.

Chapter 1 is the introduction. It contains the motivation, some historical notes, and the description of the thesis' structure.

Chapter 2 gathers a number of facts and definitions used in the thesis. Namely, it contains the definitions of NL and of its extensions with modal and classical connectives. In this chapter the author recalls known results about proof theoretic properties of NL (sequent calculus); recalls the definitions of extensions of NL with modal and classical logics and their basic properties; recalls the algebraic and relational semantics of NL; and, finally, recalls the definition of type grammars based on NL and its extensions and gives some examples, and recalls their relation to context-free grammars.

The original new results are in Chapters 3-6.

In Chapter 3, the author presents a very natural syntactical proof of the extended subformula property (Section 3.2) and provides a polynomial time decision procedure for modal NL with assumptions (Section 3.3). He also proves that the type grammars based on each of the 4-, T-, S4-, or S4- modal extension of NL are equivalent to context-free grammars (Section 3.4).

These results in Section 3.2 have been published in Logic Journal of the IGPL (LJIGPL) and the results in Sections 3.3 and 3.4 have been presented at the 4th International Conference on Language and Automata Theory and Applications (LATA 2010).

In Chapter 4, the author proves the interpolation theorem (Section 4.2), strong finite model property (SFMP) and finite embeddability property (FEP) for the class of algebras (residuated groupoids) corresponding to distributive full nonassociative Lambek calculus (DFNL) with modalities based on 4,T, and S4 (Section 4.4). Naturally, the proofs of SFMP and FEP are semantical. They are based on the subset construction with a corresponding closure operator.

SFMP immediately implies that these logics are strongly decidable (i.e., their consequence relation is decidable).

The author also proves context-freeness of the type grammars based on the above logics with finite set of assumptions (Section 4.5).

The results in Sections 4.2 and 4.4 have been published in LJIGPL and the results in Section 4.5 have been presented at the 7th International Conference on Logical Aspects of Computational Linguistics (LACL 2012).

Chapter 5 deals with the complexity of boolean full nonassociative Lambek calculus (BFNL) and its modal extensions. In this chapter, the author proves PSPACE completeness of BFNL and PSPACE hardness of the modal extensions of BFNL with either of T, 4, or S4. The proofs are based on reductions to various modal and temporal logics.

In Chapter 6, the author shows that the S5 modal extensions of NL, DFNL, and BFNL (NLS5, DFNLS5, and BFNLS5, respectively) are decidable. This follows from a much stronger result also presented in the same chapter. The author proves

- a) that the extensions of the above logics with relevant logic ( $NL_{S5}(*), DFNL_{S5}(*),$  and  $BFNL_{S5}(*),$  respectively) are conservative extensions of their “nonrelevant counterparts”  $NL_{S5}, DFNL_{S5},$  and  $BFNL_{S5}$  and
- b) that these extensions  $NL_{S5}(*), DFNL_{S5}(*),$  and  $BFNL_{S5}(*)$  are decidable.

To prove a), the author introduces sequent systems for the logics under consideration and proves cut elimination, and to prove b), he introduces algebraic models for these logics and shows that these models possess SFMP. The proof of b) is an extension of the proof of Proposition 4.4.1.

This chapter extends the results published in LJIGPL.

Chapter 7 is a short conclusion that contains a summary of the obtained results.

**Evaluation** The thesis results are new to the best of my knowledge and are of interest for researches in related areas. A part of these results has already been presented at two referred international conferences and published in a professional journal. In particular, LACL is very competitive and is one of the most prestigious conference on logic, languages, and information.

The thesis shows a good command of the author in the related research areas. It is generally well-written, though its technical parts (especially Chapter 4) are not easy for reading. However, this is what usually happens in this kind of proofs and I do not see any way to improve the writing.

It should be noted that, in this work, the author continues and extends previous research of the thesis supervisor, which is usually the case, with very few exceptions.

**Comments and suggestions** I marked quite a few corrections and suggestions (all are minor, mostly editorial) in the manuscript, which I will give directly to the author after examination.

I also have a number of questions concerning alternative proofs and other possible modal extensions of NL. I will ask these questions at the examination.

Finally, in the conclusion of a doctoral dissertation, in addition to a summary of the obtained results, one would expect an outline of directions of possible future research on the subject of the dissertation.

**Recommendation** Summing up, the submitted dissertation satisfies all the criteria I know. Thus, in my opinion, it should be accepted and the author should be awarded the doctor degree in mathematics (computer science).

