Abstract

The aim of this dissertation is to investigate the properties of the noncommutative Fréchet algebra with involution, called the algebra of smooth operators. This algebra is isomorphic as a Fréchet space to the commutative algebra *s* of rapidly decreasing sequences (isomorphic also to the well-known Schwartz space of smooth rapidly decreasing functions), and thus it is a kind of noncommutative analogue of the algebra *s*.

A significant part of the dissertation is devoted to the description and classification of the closed commutative *-subalgebras of the algebra of smooth operators. For instance, we show that such a subalgebra is isomorphic to a closed *-subalgebra of the algebra *s* if and only if it is isomorphic (as a Fréchet space) to a complemented subspace of *s*. We also find the multplier algebra of the algebra of smooth operators, prove theorems on spectral and Schmidt representations of elements of this algebra and show that there is a Hölder continuous functional calculus for normal smooth operators. Most of the proofs are based on the theory of bounded and unbounded operators on a Hilbert space and the theory of nuclear Fréchet spaces.