Abstract

The dissertation “On certain semigroups of convex compact sets and minimal representations of elements of their quotient spaces” invistigate a semigroup polyhedrons with prescribed face directions.

Chapter 1 describes the basic notions used in the following chapters. It consists of two paragraphs. The first paragraph is intended to determine symbols and signs which will differ from the formal designations used in the literature. Paragraph 1.2 introduces the basic, commonly known definitions due to the ambiguity of some of them.

In Chapter 2, we introduce definitions and theorems of convex analysis referring to the family of nonempty, bounded, closed and convex sets. This chapter consists of four paragraphs. The purpose of Paragraph 2.1 is to introduce into a family the sets of nonempty, bounded, closed and convex semigroup structure and even an abstract convex cone structure. We also show that in this semigroup, the order law of cancellation is satisfied. In Paragraph 2.2, we introduce the equivalence relation in the Cartesian product of the above semigroups. We construct the Minkowski-Rådström-Hörmander space, which is a quotient space relative to this relation. We show that this space is a vector space and we define a partial order on it. We are also considering elements of this space called virtual bodies. These equivalence classes are used in quasidifferential calculus. The quasidifferential of function is an element of the MRH space. An important issue of the quasidifferential calculus is to find a minimal representation of the quasidifferential, which is equivalent to indicating the minimal element in relation to the partial order within the virtual body of nonempty, compact and convex sets. In Paragraph 2.3, we formulate methods for the reduction of pair of sets. Next, we illustrate these methods on the example in a two-dimensional real space. The purpose of Paragraph 2.4 is to discuss the known criteria of minimality for nonempty, compact and convex pairs. We formulate theorems which are characterized minimality of pairs for one-dimensional and two-dimensional real space. Due to the fact that these criteria can not be applied in higher dimensions, Paragraph 2.4 ends with open questions regarding the minimality. These questions are the main motivation of this dissertation.

In Chapter 3 we introduce to the theory of polyhedrons with prescribed face directions, called G-polyhedra, and repeating the results from Chapter 2 for the G-polyhedra family. This chapter consists of six paragraphs. In Paragraph 3.1, we indicate the motivations to study this family. In Paragraph 3.2, we define the basic notions related to the theory of G-polyherda. We also point out the reason why this family does not have, generally, the structure of the semigroup with the Minkowski sum. Paragraph 3.3 is intended to discuss the representation of polygons and introducing analogous representations for G-polygons. These representations are widely used in

the further part of this chapter due to presented formulas used to transform one representation into another. In Paragraph 3.4, we define a modified Minkowski sum, which will be used to define a semigroup structure in a family of G-polyhedra. Also in this paragraph we prove this structure. In Paragraph 3.5, we define the relation of equivalence and the quotient space. We can, after introducing a partial order, consider the methods of reduction and the criteria of minimality, as in Chapter 2. Paragraph 3.6 examines this issue in the two-dimensional case.

In Chapter 4, we decribe assumptions, which have to be satisfied by the family of G-polyhedra with Minkowski sum to be a semigroup. This chapter consists of four paragraphs. In Paragraph 4.1 we discuss the issue and we reformulate it to the problem of the so-called skeletons. In Paragraph 4.2, we resolve the geometric problem of selecting sets with an internally intersecting skeleton on a plane. This is not directly related to the family of G-polyhedra, but the statements developed in this paragraph are applicable in the next paragraph. Paragraph 4.3 has a structure analogous to the previous paragraph. We define the problem and introduce notions connected with its solution, and then we prove the theorems determining the structures of sets with the internally intersecting spherical skeleton. These theorems have a direct relationship with the problem of the semigroup structure of family of G-polyhedra. In Paragraph 4.4, we point to specific families of G-polyhedra, for whom it is not necessary to modify Minkowski sum.

In Chapter 5, we examine pairs of G-polyhedra for their G-minimality. This chapter has three paragraphs. The notion of G-minimality is significantly distinct from the minimality, because the pair G-polyhedra, as a pair of nonempty, compact and convex sets, can be reduced to a minimal pair, which is not a pair of G-polyhedra. Paragraph 5.1 contains questions similar to the questions at the end of Chapter 2. Also in this paragraph we use linear programming methods to define reduction methods and the criterion of G-minimality, which is the answer to first two, previously formulated, questions. Paragraph 5.2 contains a summary, in the form of an algorithm, of the considerations contained in the previous paragraph. This algorithm is the answer to the last of the formulated questions. In Paragraph 5.3, we present the example of using an algorithm to indicate the set of all G-minimal pairs equivalent to the fixed pair of G-polyhedra.

In Chapter 6, we point to certain applications of results of this dissertation. This chapter consists of two paragraphs. In Paragraph 6.1, we formulate statements that we can use G-minimality to confirmation the minimality of pair of polyhedra. Paragraph 6.2 concerns other application of the G-polyhedra theory and indicates future research directions.

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