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# On Reeb graphs and related objects

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This thesis presents results concerned with Reeb graphs of smooth functions on manifolds. One of the fundamental problems in this topic, investigated also by other authors, is to distinguish and characterize graphs which could be the Reeb graphs of functions from a given class. More difficult problem is a question on the set of possible Reeb graphs of functions on a fixed manifold. In this way from a function invariant we obtain features describing a manifold itself. We use Morse theory and combinatorial methods to provide a series of realization theorems for Reeb graphs, starting from determining possible cycle ranks of Reeb graphs and ending with the description of their homeomorphism or isomorphism types.

Two kinds of objects are naturally associated with Reeb graphs: epimorphisms onto free groups, called the Reeb epimorphisms, which are induced on fundamental groups by the quotient maps from manifolds to Reeb graphs, and systems of hypersurfaces in manifolds, corresponding to edges in Reeb graphs outside spanning trees. We present number of properties of these objects and their connections with Reeb graphs. In particular, we prove that any epimorphism from the fundamental group of manifold onto free group is induced by a system of hypersurfaces non-separating the manifold. We also show the relationship between cobordism classes of systems of hypersurfaces modulo diffeomorphisms of manifold and strong equivalence classes of epimorphisms onto free groups. The full computation of these classes is made for surfaces. The obtained results allow us to extend realization theorems to characterize not only Reeb graphs but also the Reeb epimorphisms of Morse functions. In the case of surfaces we provide complete characterization of Reeb epimorphisms of simple Morse functions and we show their utility in topological conjugacy of functions.

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