Summary

This thesis is concerned with full nonassociative Lambek calculus with different modalities which is a family of substructural logics. Substructural logics are logics which omit some structural rules, e.g. contraction, weakening, commutativity. Nonassociative Lambek calulus (NL) introduced by Lambek is a propositional logic omitting all structural rules, which can be treated as a basic core of substructural logics. NL can be enriched in different ways. One affixes additive connectives $\wedge$, $\vee$ and optionally constants $\bot$, $\top$ and classical negation, which yields full nonassociative Lambek calculus (FNL) and its extensions DFNL and BFNL, satisfying the distributive laws for $\wedge$, $\vee$ or the laws boolean algebras, respectively. Modal nonassociative Lambek calculus (NL$\Diamond$) is NL enriched with modalities $\Diamond$, $\Box^{\downarrow}$ satisfying the residuation law; $\Diamond A \Rightarrow B$ iff $A \Rightarrow \Box^{\downarrow} B$, which enables one to use certain structure postulates in a controlled way.

In this thesis we study extensions of NL, DFNL, and BFNL enriched with modal operators admitting any combinations of basic modal axioms (T), (4) and (5). We also consider assumptions. In other words, we study consequence relations of these logics. We prove and use some basic proof-theoretic properties of them, e.g. cut elimination, interpolation, subformula property. We also consider algebras corresponding to these logics. We prove strong finite model property (SFMP) for some classes of algebras, which yields the decidability of consequence relations. Moreover we study the complexity of these logics (and consequence relations). We construct some decision procedures for the additive free logics considered in this thesis. It turns out that the complexity of these logics (and consequence relations) are P (polynomial time). Further we show that BFNL is PSPACE-complete and its modal extensions are PSPACE-hard.

Grammars based on logics of types are called type grammars. A characteristic feature of type grammars is the usage of (logical) types

as carriers of grammatical information. We focus on type grammars based on modal extensions of NL and DFNL. We show that the type grammars based on some logics, considered in this thesis, enriched with assumptions, generate context-free languages.

Cin Zhe.

林鑫