

NUMBERS OF THE FIBONACCI TYPE, THEIR PROPERTIES AND
APPLICATIONS TO THE COUNTING PROBLEMS IN GRAPHS

Natalia Bednarz

Abstract

The main aim of this thesis is to study numbers of the Fibonacci type, in particular their applications in graphs. The first part of the thesis presents results concerning the number of all $(A, 2B)$ -edge colourings in unicyclic graphs. The presented results are a continuation of the research on the $(A, 2B)$ -index in trees. Using the properties of Fibonacci numbers, Lucas numbers and telephone numbers, the lower bound and the upper bound of the $(A, 2B)$ -index in unicyclic graphs are determined. We provided full characteristic of graphs achieving these extreme values. Moreover, we determined the successive extreme unicyclic graphs with respect to the number of all $(A, 2B)$ -edge colourings. For special classes of unicyclic graphs the exact value of the $(A, 2B)$ -index is determined, which resulted in obtaining new identities for Fibonacci and Lucas numbers.

In the second part of the thesis we introduced and studied a new two-parameter generalization of the numbers of the Fibonacci type, named (k, p) -Fibonacci numbers, which generalizes Fibonacci numbers, Pell numbers and Narayana numbers, simultaneously. We presented the properties of the (k, p) -Fibonacci numbers, including their matrix representation. We gave some combinatorial interpretations of these numbers. Moreover, using these interpretations, we proved identities for the (k, p) -Fibonacci numbers. The results included in this chapter generalize, among others, the results presented in Falcón S. et al. (2007), Koshy T. (2001) and (2014), Kwanik M. et al. (2000), Ramirez J. L. et al. (2015) and Stakhov A. P. (1977).

A significant part of the thesis has been published or is in print in the following papers:

- [A] Bednarz N., Włoch A., Włoch I., *The Fibonacci numbers in edge coloured unicyclic graphs*, *Utilitas Mathematica* 106 (2018), 39-49,
- [B] Bednarz N., Włoch I., *Some interpretations of the (k, p) -Fibonacci numbers*, *Commentationes Mathematicae Universitatis Carolinae*, (in print).

Natalia Bednarz