On isometries and Tingley's problem for combinatorial Tsirelson spaces

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In 1987, Tingley proposed a question that has since become known as Tingley's problem:

Let X and Y be normed spaces with unit spheres $\mathbb{S}_X$ and $\mathbb{S}_Y$, respectively. Suppose that $U\colon \mathbb{S}_X \to \mathbb{S}_Y$ is a surjective isometry. Is there a linear isometry $\tilde{U}\colon X \to Y$ such that $\tilde{U}|_{\mathbb{S}_X} = U$?

This problem has been positively resolved for certain classical Banach spaces, including $\ell_p(\Gamma)$, $L_p(\mu)$ $(1 \leqslant p \leqslant \infty)$, and $C(\Omega)$. Recently, a positive solution was also found for 2-dimensional Banach spaces, but the question remains open in higher dimensions.

In this talk, we will extend the existing results on surjective isometries of unit spheres in the Tsirelson space $T\left[\frac{1}{2}, \mathcal{S}_1\right]$ to the class of combinatorial Tsirelson spaces $T[\theta, \mathcal{S}_\alpha]$ for any integer $\theta^{-1} \geq 2$ and $1 \leqslant \alpha < \omega_1$, where $\mathcal{S}_\alpha$ denotes the Schreier family of order α. Additionally, we will characterize the linear isometries of these spaces.

Our findings provide an affirmative answer to Tingley's problem within the class of combinatorial Tsirelson spaces under the conditions specified.

References

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