

Structure constants of Jack characters

Adam Burchardt

Abstract

In 1996 Goulden and Jackson introduced a family of coefficients $(c_{\mu,\nu}^\lambda)$ indexed by triples of partitions which arise in the power sum expansion of some Cauchy sum for Jack symmetric functions $J_\pi^{(\alpha)}$. Goulden and Jackson suggested that there is a combinatorics of matchings hidden behind the coefficients $c_{\mu,\nu}^\lambda$. This *Matchings-Jack Conjecture* remains open.

Jack characters are a generalization of the characters of the symmetric groups, they provide a kind of dual information about the Jack polynomials. We investigate the structure constants $g_{\mu,\nu}^\lambda$ for Jack characters. They are a generalization of the connection coefficients for the symmetric groups. We give formulas for the top-degree part of $g_{\mu,\nu}^\lambda$ and $c_{\mu,\nu}^\lambda$. We present those results in context of Matchings-Jack Conjecture of Goulden and Jackson.

We adapt the probabilistic concept of cumulants to the setup of a linear space equipped with two multiplication structures. We present an algebraic formula which expresses a given nested product with respect to those two multiplications as a sum of products of the cumulants. This formula leads to some conclusions about the structure constants of Jack characters. We also show that our formula may be understood as an analogue of Leonov-Shiraev's formula.

Poznań, 16 marca 2018

Adam Burchardt