Abstract

In 1965 Erdős asked what is the maximum number of edges in k-uniform hypergraphs on n vertices in which the largest matching has s edges. He conjectured that it is maximized either for cliques, or for graphs which consist of all edges intersecting a set of s vertices. Neither construction is uniformly better than the other in the whole range of parameter s ($1 \leq s \leq n/k$), so the conjectured bound is the maximum of these two possibilities.

In this thesis we present results obtained while working on this problem. In particular, we confirm Erdős' conjecture in a general k-uniform case for $n \geq 2k^2 s/\log k$, and, more importantly, settle it in the affirmative for $k = 3$ and n large enough. We also derive a stability result which shows that in order to verify Erdős' conjecture it is enough to prove it in an asymptotic form.

In the last chapter, we discuss new conjectures and results obtained while working on Erdős' problem. In particular, we formulate a structural conjecture that might be considered as an asymptotic generalization of Tutte's Theorem for hypergraphs, and, if true, may bring us closer to solve the Erdős' matching problem. Moreover, we state a new probabilistic conjecture on small deviation inequalities, of a similar flavour as Samuels' conjecture stated in 1965. We confirm it in a few instances, by proving that it is asymptotically equivalent to the fractional version of Erdős' matching problem.