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ABSTRACT

My doctoral dissertation "Solution of the invariant subspace problem for non-Archimedean Köthe spaces" solves the problem of the existence of linear and continuous operators on some non-Archimedean Köthe spaces that have no nontrivial closed invariant subspaces. This thesis is based on my published work H. Kasprzak, The invariant subspace problem for non-Archimedean Köthe spaces. *J. Math. Anal. Appl.* **453** (2017), no. 2, 1086-1110 and is its refinement. This refinement consists of the following three pillars: using the function f with real values, duality and algebraization. The function $f\colon \mathbb{N} \to \mathbb{R}$ plays a key role in the dissertation. By means of this function, we define a continuous linear operator $T_0\colon \Lambda_0(A) \to \Lambda_0(A)$ on a dense subspace $\Lambda_0(A) = \mathrm{Lin}\{e_1, e_2, \dots\}$ of a non-Archimedean Köthe space $\Lambda(A)$. In my published work, the function f is defined with integer values. In the doctoral thesis we use the function f with real values, to directly define the operator T_0 we use the function $\hat{f}\colon \mathbb{N} \to \mathbb{N},\ n \mapsto [f(n)]$. This gives large facilities. Among other things, it allows to create a duality. The duality means that the function f can be replaced with any other function, but the matrix coefficients of a given non-Archimedean Köthe space mast change accordingly. Using the function $f \equiv 0$ we get a simple form of the operator T_0, and using the function f other than zero it can be easier to use geometric properties of a considered non-Archimedean Köthe space. Algebraization is based on the fact that in one aspect the operator T_0 can be defined completely arbitrarily and the correctness of Lemma 1.6 is obtained automatically.

In the first chapter, we formulate and prove the General Theorem. The General Theorem is not one theorem, but a scheme of many theorems. In the second chapter, we define and prove two specific schemes, which are

consequences of the General Theorem. These schemes are easily proved for the function $f \equiv 0$, and then are easily generalized for any function f. And in the third chapter, we show that non-Archimedean Köthe spaces fulfilling certain properties are isomorphic with the schemes from the second part.

In the first paragraph of the first chapter, we formulate the assumptions of the General Theorem. This theorem says that if for a given function $f : \mathbb{N} \to \mathbb{R}$ a linear operator $T_0 \colon \Lambda_0(A) \to \Lambda_0(A)$, $a^{\hat{f}(n)} e_n \mapsto \sum_{i=1}^{n} a^{\hat{f}(i)} e_i + a^{\hat{f}(n+1)} e_{n+1}$ is defined that fulfills these assumptions, then T_0 is continuous and extends to a linear and continuous operator $T \colon \Lambda(A) \to \Lambda(A)$ that has no nontrivial closed invariant subspaces. These assumptions are defined so that the duality can be used.

In the second paragraph we assume that $f \equiv 0$ and we prove a few lemmas needed to prove Theorem 1.9 from the third paragraph, which is a special case of the General Theorem for $f \equiv 0$.

By using Theorem 1.11, we create the duality. This theorem says how an operator $T_0 \colon \Lambda_0(A) \to \Lambda_0(A)$ defined by a function f can be replaced with an operator $R_0 \colon \Lambda_0(B) \to \Lambda_0(B)$ defined by any other function g, with the spaces $\Lambda(A)$ and $\Lambda(B)$ being isomorphic. From Theorem 1.9 and Theorem 1.11 we derive the General Theorem, i.e. Theorem 1.12. With the assertion proved for $f \equiv 0$ and the fact that f can be replaced with any other function, we get the General Theorem.

In the first paragraph of the second chapter, we define and prove the first scheme for $f \equiv 0$, which then generalize for any f in the second paragraph. In the third paragraph, we introduce the second scheme and its generalization.

In the first paragraph of the third chapter, we prove the basic theorem proved in my published work, using the first scheme for $f \equiv 0$. In the second paragraph, we use the first scheme for any f, and we do not assume that the matrixes coefficients of non-Archimedean Köthe spaces are bounded from below.

In the further part of the third chapter, we assume that the coefficients of matrixes of non-Archimedean Köthe spaces are limited from below, so we can use the second scheme. The division of non-Archimedean Köthe spaces due to the type of the coefficients is not strict and depends to a large extent on the function f.

From Theorem 3.6, we get successively theorems 3.9, 3.10 and 3.11. Theorem 3.11 generalizes Theorem 3.8 from my published work. And from Theorem 3.11 we obtain successively theorems 3.14, 3.15, 3.16 and 3.17. Theorem 3.15 generalizes Theorem 4.1 from my work for nuclear non-Archimedean

Köthe spaces. And Theorem 3.17 generalizes the results for non-Archimedean analytic functions proved in this work.