# SEPARABLE REDUCTIONS, RICH FAMILIES AND PROJECTIONAL SKELETONS IN NON-SEPARABLE BANACH SPACES

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A Banach space $X$ is isomorphic to a Hilbert space if (and only if) every separable subspace $Y \subset X$ is such. Similarly, $X$ is reflexive if (and only if) every separable subspace of it is reflexive. The Radon Nikodým property (which is equivalent to dentability) is separably reducible. The Asplund property of a space is also separably reducible, ...

We will also separably reduce properties of functions, like boundedness, continuity, (sub)differentiability, ... E.g. a function $f : X \to \mathbb{R}$, there is a cofinal family $\mathcal{S}(X)$ of separable subspaces of $X$ such that for every $x \in Y \in \mathcal{S}(X)$: $f$ is continuous at $x$ if (and only if) $f|_Y$ is continuous at $x$. By strengthening this, we arrive at the concept of *rich family* — meaning a family which is cofinal and in addition $\sigma$-closed. Rich families are very useful in grasping non-separable Banach spaces. They can serve as an index set for the so called projectional skeletons (PS), introduced in 2009 by W. Kubiś. PS is a modern substitute of the classical Lindenstrauss' projectional resolution of identity (PRI). Of course, from PS one can construct PRI. Several big classes of non-separable Banach spaces can be characterized by presence of a PS with some extra properties.