

OPTIMAL GAGLIARDO–NIRENBERG INEQUALITY IN R.I. SPACES

KAROL LEŚNIK

I will discuss the Gagliardo–Nirenberg inequality

$$(1) \quad \|\nabla^j u\|_Z \leq C \|\nabla^k u\|_X^{j/k} \|u\|_Y^{1-j/k}, \quad 1 \leq j \leq k,$$

in the context of rearrangement invariant Banach function spaces.

Firstly, I will prove that (1) holds for $Z = X^{1/k}Y^{1-j/k}$ without any additional assumptions on r.i. Banach spaces X, Y . In this direction the proof requires a new approach that works in all r.i. spaces and may be regarded as a kind of sparse domination method. As a byproduct it gives also a new pointwise estimates for derivatives of independent interest. Finally, I will explain that such choice of Z is actually optimal for a wide class of r.i. spaces.

The talk is based on a joint work with Tomas Roskovec and Filip Soudsky.