

The linear topological invariant (Ω) for kernels of partial differential operators and its application to Runge type approximation theorems

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Abstract:

A well known consequence of Runge's classical theorem on rational approximation is that for open subsets $X_1 \subseteq X_2$ of the complex plane \mathbb{C} , every holomorphic function on X_1 can be approximated by a holomorphic function on X_2 with respect to the compact-open topology, precisely when X_2 does not contain a compact component of $\mathbb{C} \setminus X_1$. In the 1950s, this approximation theorem has been generalized independently by Lax [4] and Malgrange [5] from holomorphic functions, i.e. functions in the kernel of the Cauchy-Riemann operator, to kernels of elliptic constant coefficient differential operators.

Recently, there has been an renewed interest in such approximation results, namely in so-called quantitative Runge type approximation results for elliptic operators of second order on the one hand (cf. [6]) and generalizations of the Lax-Malgrange Theorem to certain classes of non-elliptic operators on the other hand (cf. [1], [2], [3]).

In our talk, we show how recent results on Vogt's and Wagner's topological invariant (Ω) , introduced in [7], for kernels of partial differential operators can be used to obtain quantitative Runge type approximation results for elliptic operators of arbitrary order, non-degenerate parabolic operators and the one dimensional wave operator.

The talk is based on joint work with A. Debrouwere (University of Brussels).

References

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