

ABELIAN VARIETIES OVER p -ADIC FIELDS
ROZMAITOŚCI ABELOWE NAD CIAŁAMI p -ADYCZNYMI
PRACA DOKTORSKA

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In the thesis we study three problems related to arithmetic of abelian varieties over p -adic fields. The first part of the thesis studies the arithmetic complexity of p -torsion of an abelian variety over the field of p -adic numbers. This is connected to an unproven conjecture of David and Weston from 2008. We establish a relation between this problem and the notion of the canonical lift of an abelian variety. We also try to verify this conjecture for elliptic curves with complex multiplication, which leads to looking for primes in some recurrence sequences.

In the next part of the thesis we investigate the equivariant behaviour of the Hodge–de Rham exact sequence of a curve with an action of a finite group in positive characteristic. We show that if its Hodge–de Rham sequence splits equivariantly then the group action is weakly ramified. We also discuss converse statements and link this problem to lifting coverings of curves to the ring of Witt vectors of length 2. This allows us to exhibit new examples of abelian varieties without canonical lifts.

In the last part of the thesis we are concerned with the division fields of abelian varieties defined over number fields. Using Kummer theory of abelian varieties and various p -adic methods (such as the theory of Néron models and the classification theorem for compact p -adic Lie groups), we prove a lower bound on the class numbers of the division fields. This lower bound depends on the Mordell–Weil rank of A and the reduction of p -torsion points modulo primes above p .

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