# Abstract

The main goal of this thesis is to examine solutions of linear differential equation

$$y'(x) = \lambda y(x) + f(x), \quad (\lambda \neq 0),$$

where $f$ is an almost periodic function in a generalized sense. We consider two classes of generalized almost periodic functions - the class of almost periodic functions in view of the Lebesgue measure (briefly: $\mu$-a.p.) and the class of almost periodic functions in the Levitan sense (briefly: LAP). For both classes we give conditions which guarantee the existence and the nonexistence of a generalized almost periodic solution, respectively. Since a solution of the above equation usually can be expressed by means of the convolution with some function from the space $L^1(\mathbb{R})$, so a special attention is devoted by the convolution operator. We try to generalize results known from the literature for the convolution operator defined on the space of almost periodic functions in the Stepanov sense and on the space of bounded LAP functions.

Moreover, in this dissertation we consider the so-called *„leaky-integrate-and-fire"* model. This model is defined for the same linear differential equation ($\lambda \leqslant 0$) with additional condition

$$\text{if} \quad y(x_0) = 1, \quad \text{then} \quad \lim_{x \to x_0^+} y(x) = 0,$$

where $y$ is a solution of the above differential equation and $f$ is a generalized almost periodic function. In this model we investigate properties of the *„firing map"* defined by the formula

$$\Phi(t) = \inf\Big\{ s > t \colon e^{\sigma t} = \int_t^s (f(u) - \sigma) e^{\sigma u} du \Big\}$$

and properties of the *„displacement map"* defined by $\Psi(t) = \Phi(t) - t$. Moreover we consider the so-called *„rotation number"*, defined by

$$\rho(f) = \lim_{n \to \infty} \frac{\Phi^n(t_0)}{n},$$

where $\Phi^1(t_0) = \Phi(t_0)$, $\Phi^n(t_0) = \Phi(\Phi^{n-1}(t_0))$, for $n > 1$, and $t_0 \in \mathbb{R}$. We try to extend results known for the class of Stepanov almost periodic functions on the class of $\mu$-a.p. functions.

Moreover, in this thesis we compare the class of continuous $\mu$-a.p. functions with the class of LAP functions. Simultaneously, we give an answer to some open question concerning almost automorphic functions.

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