Integral operators and multipliers between spaces of holomorphic functions in the unit disc

Abstract of doctoral dissertation

The main aim of the dissertation is to present a variety of results concerning some classes of operators defined on spaces of holomorphic functions in the unit disc of the complex plane. We start with analytic integral operators. Due to their important applications, we study Volterra and Libera operators defined on Hardy spaces. We describe the optimal domain for these operators. Furthermore, we investigate stability property for the discrete Hardy operator defined on the positive solid hull of Hardy spaces.

Next, we proceed to some problems concerning analytic multiplication operator. We examine it from both sequence (Hadamard product) and classical point of view. We are mainly interested in characterizing spaces of coefficient and function multipliers between some general classes of spaces of holomorphic functions in the unit disc. We examine function multipliers for spaces of holomorphic functions generated by symmetric sequence spaces. In particular, we prove that with some conditions imposed on a generating space the space of multipliers is trivial and check how Köthe duality affects its description.

The next topic pertains to the theory of abstract Nevanlinna spaces. We prove their basic topological properties and consider the essential problem of inner-outer factorisation, analogous to the classical one given by F. Riesz for Hardy spaces. We characterize spaces of coefficient multipliers between abstract Nevanlinna spaces and Hardy spaces. Using this result, we describe the dual space for abstract Nevanlinna spaces. We also characterize the solid hull of Privalov spaces, which are classical examples of investigated abstract Nevanlinna spaces.

In the dissertation, along with some results concerning the theory of operators, we use some classical facts from the theory of holomorphic functions together with the theory of harmonic and subharmonic functions. Thus the subject of the dissertation balances on the edge of those three important areas of mathematical analysis.

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