Topological selections

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In 1924, Menger observed that any metric space X which is *σ-compact* (i.e., it is a countable union of its compact subsets) has such a property that for any basis $\mathcal{B}$ of X, there are sets $B_0, B_1, \ldots \in \mathcal{B}$, such that $\lim_{n\to\infty}\mathrm{diam}(B_n) = 0$ and $X = \bigcup_{n\in\omega} B_n$. Menger conjectured that the above property characterizes σ-compactness in the class of metric spaces. Soon thereafter Hurewicz reformulated the Menger property without using a metric: for any sequence $\mathcal{U}_0, \mathcal{U}_1, \ldots$ of open covers of a given topological space, there are finite sets $\mathcal{F}_1 \subseteq \mathcal{U}_0, \mathcal{F}_1 \subseteq \mathcal{U}_1, \ldots$ such that the family $\bigcup_{n\in\omega} \mathcal{F}_n$ is an open cover of the space. In that way, the definition of the Menger property was extended on all topological spaces. By the results of Fremlin–Miller and Bartoszyński–Tsaban, there is in ZFC a subspace of the real line which is Menger but no σ-compact.

The aim of the talk is to present an overview of the Menger property which is one of the most influential property in the topological selections theory and it has many connections to topology, set-theory and function spaces. We also present some recent results related to this topic.

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