

## Abstract

The aim of this thesis is to investigate several properties of composition operators and weighted composition operators on the space of smooth functions, i.e., operators of the form  $C_\psi : C^\infty(\Omega) \rightarrow C^\infty(\Omega)$ ,  $F \mapsto F \circ \psi$ , or of the form  $C_{w,\psi} : C^\infty(\Omega) \rightarrow C^\infty(\Omega)$ ,  $F \mapsto w \cdot (F \circ \psi)$ , where  $\Omega \subset \mathbb{R}^d$  is open,  $C^\infty(\Omega)$  is the Fréchet space of smooth functions on  $\Omega$ , and the functions  $\psi : \Omega \rightarrow \Omega$ ,  $w : \Omega \rightarrow \mathbb{C}$  are smooth. Those operators are very natural examples of operators acting on the space of smooth functions, which is very important in analysis.

The first part of this dissertation is devoted to the question of for which smooth symbols  $\psi : \mathbb{R} \rightarrow \mathbb{R}$  the range of the composition operator  $C_\psi$  is closed in  $C^\infty(\mathbb{R})$ . We present several necessary and sufficient conditions for this property. In particular, we prove that if  $\psi : \mathbb{R} \rightarrow \mathbb{R}$  is a smooth semiproper function which has no flat points, then the range of  $C_\psi$  is closed.

The second part of this dissertation is devoted to the study of several dynamical properties of composition operators and weighted composition operators acting on  $C^\infty(\Omega)$ , where  $\Omega \subset \mathbb{R}^d$  is open. We characterize hypercyclic (in case if the weight is real valued), weakly mixing, and mixing weighted composition operators. As a by-product we obtain a characterization of hypercyclic, weakly mixing, and mixing composition operators. Then we show that those three classes of operators coincide in the one-dimensional case.

Prostački