SCALING A THEOREM OF HARALD BOHR

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We discuss recent joint work with Ingo Schoolmann on general Dirichlet series.

Given a frequency $\lambda = (\lambda_n)$, that is a strictly increasing sequence of non-negative real numbers, a λ-Dirichlet series is a series of the form $D = \sum a_n e^{-\lambda_n s}$, where s is a complex variable and (a_n) the sequence of complex coefficients. To recall two prominent examples, observe that the choice $\lambda = (\log n)$ leads to ordinary Dirichlet series $\sum a_n n^{-s}$, whereas the choice $\lambda = (n)$ after the substitution $z = e^{-s}$ generates power series $\sum a_n z^n$ in one variable.

The study of general Dirichlet series in fact was one of the hot topics in mathematics at the beginning of the 20th century. Among others, H. Bohr, Hardy, Landau, and M. Riesz were the leading mathematicians in this issue.

Our research is part of a general attempt to establish a modern theory of such series. Modern in the sense that we try to use tools and techniques offered by today active areas like functional, harmonic and complex analysis or analytical number and probability theory.

A fundamental result of H. Bohr shows that under a certain condition on the frequency $\lambda = (\lambda_n)$ (preventing the λ_n's from getting too close too fast), every Dirichlet series $D = \sum a_n e^{-\lambda_n s}$ converges uniformly on all half-planes $[Re > \sigma]$, $\sigma > 0$, provided D is pointwise convergent on some half-plane and has a limit function extending to a bounded holomorphic function f on $[Re > 0]$.

We intend to discuss several extensions of this result - and the idea is to touch this way many of the new key elements of our general approach.

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