Lattice Lipschitz diagonal maps on Banach function spaces

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Abstract. A lattice Lipschitz operator on Euclidean spaces is a function $T : \mathbb{R}^n \to \mathbb{R}^n$ that satisfies that $|Tv - Tw| \le K \cdot |v - w|$ for all $v, w \in \mathbb{R}^n$ for a certain constant $K > 0$, where the standard order is considered. It can be easily proved that this requirement implies the diagonal nature of the map, although linearity is not necessarily required. Recently, these operators have been introduced in the context of mathematical tools for machine learning, where they have proven to be quite useful. This is largely because, despite being vector-valued, they satisfy a McShane-Whitney extension theorem.

In this talk, we will explore the generalization of this class of operators to those between Banach function spaces, which presents challenging problems compared to the linear case, where the associated maps are classical point-wise multiplication operators. These lattice Lipschitz operators represent a specific class of superposition operators. The motivation for this research lies in the potential to develop a Lipschitz factorization theory as rich as that known for the linear case, thus leading to a Lipschitz adaptation of classical factorization theorems.