Sobolev embeddings and $(\Phi, 1)$-summing operators

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The talk will be devoted to the study of absolutely summing properties of the Sobolev embedding operator. Recall that a continuous linear operator $T : X \to Y$ between Banach sequence spaces is said to be $(\Phi, 1)$-*summing* if there exists a constant $C > 0$ such that for all $x_1, ..., x_n \in X$,

$$\|(\, ||Tx_i||_Y \,)_{i=1}^n\|_{\ell_\Phi} \leq C \sup_{x^* \in B_{X^*}} \sum_{i=1}^n |x^*(x_i)|,$$

where B_{X^*} denotes a unit ball in the dual space of X.

We know that Sobolev embedding operator is $(p, 1)$-summing for $p > p_0 = \max\{\frac{2d}{2k+d}, p\}$. I will show how from the values of $(p, 1)$-summing norms and extrapolation methods we obtain that the Sobolev embedding is $(\Phi, 1)$-summing.

Theorem. *Let* $d \in \mathbb{N} \setminus \{0, 1\}$, $k \in \{1, 2, \ldots, d-1\}$, $1 \leq p < 2$ *and* $p < d/k$. *Then the Sobolev embedding* $S_{d,k,p} : W^{k,p}(\mathbb{T}^d) \to L_s(\mathbb{T}^d)$, *where* $s = 1/p - k/d$, *is* $(\Phi, 1)$-*summing operator for* $\Phi(x) = \frac{x^{p_0}}{|\ln(x)|^\gamma}$ *and* $0 \leq x \leq \frac{1}{2}$, *where* $p_0 = \max\{\frac{2d}{2k+d}, p\}$, $\gamma > p_0(\frac{2}{p} - 1)$.

The talk is based on a joint work with A. Pawlewicz and M. Wojciechowski.