Boundedness of the Riesz potential in central Morrey–Orlicz spaces

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The Riesz potential I_α, $0 < \alpha < n$, defined for $x \in \mathbb{R}^n$ by

$$I_\alpha f(x) = \int\limits_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy,$$

plays a significant role role in various branches of analysis, including potential theory, harmonic analysis, and partial differential equations. The study of its boundedness between L^p-spaces was initiated by S. Sobolev in 1938. In the framework of Orlicz spaces, boundedness results were first established by I. B. Simonenko in 1964 and later extended to full generality by A. Cianchi in 1999. For Morrey spaces, the first result on the boundedness of the Riesz potential was obtained by S. Spanne and published by J. Peetre in 1969. A stronger result followed in 1975, due to D. R. Adams. Since then, many authors have investigated the boundedness properties of potential type operators in various function spaces.

In this talk, we briefly review key results concerning the boundedness of the Riesz potential in Morrey type spaces. In particular, we discuss why Adams type result does not hold in central Morrey spaces, where only Spanne–Peetre type estimates can be obtained. We then present our recent results on the boundedness of the Riesz potential in central Morrey–Orlicz spaces. Specifically, we establish a Spanne–Peetre type result in this framework and construct an example illustrating the failure of Adams type result for I_α in central Morrey–Orlicz spaces under certain conditions on the Orlicz functions defining these spaces.

The talk is based on joint papers with L. Maligranda and K. Matsuoka.

The research visit to Poznań was funded by the G. S. Magnuson Foundation of The Royal Swedish Academy of Sciences (grant no. MG2023-0073).