Abstract of the doctoral thesis "Inequalities for Sums of Random Variables: a combinatorial perspective"

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We investigate bounds for the probability $\mathbb{P}(S \in I)$, where $S = X_1 + \cdots + X_n$ is a sum of independent or weakly dependent random variables and I is an interval (bounded or unbounded). We consider three rather different problems.

The first one is about the concentration of a random variable $f(X_1, \cdots, X_n)$, where $X_1, \cdots, X_n$ are independent Bernoulli random variables with parameter p and function $f : \{0,1\}^n \to \mathbb{R}$ is Lipschitz with respect to the Hamming distance. We obtain a simple bound for $\mathbb{P}(f - \mathbb{E} f \geqslant x)$, which in the given setting is better than the well-known McDiarmid inequality. From our bound we derive an isoperimetric inequality which is similar to an inequality by Talagrand obtained by a different method.

In the second part we obtain optimal bounds for $\mathbb{P}(S \in I)$, when X_i's are independent, symmetrically distributed and bounded in absolute value by 1. Our results cover the cases when $I = [x, \infty)$ or $I = \{x\}$.

The third direction of the thesis focuses on the number X_G of copies of a fixed graph G in the random graph $\mathbb{G}(n, p)$. For certain graphs G we obtain several exponential bounds for $\mathbb{P}(X_G \geqslant t\mathbb{E} X_G)$ which are optimal up the constant in the exponent.